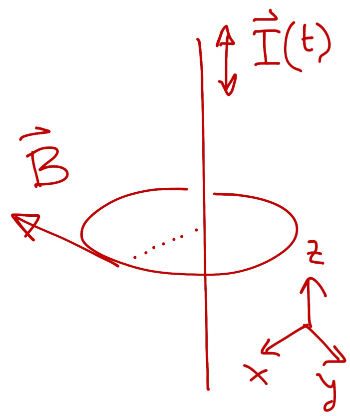
Midterm Exam : Waves & Optics

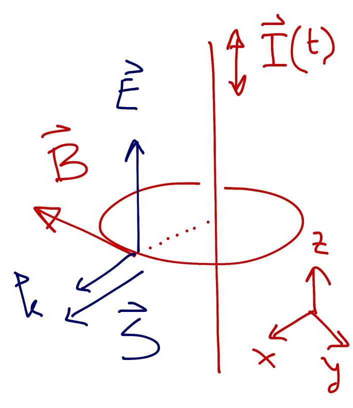
12 December 2016, 9:00-11:00

- Put your name and student number on each answer sheet.
- Answer all questions short and to the point, but complete; write legible.
- Final point grade = total number of points/4 + 1

1. **AC Wires (12 points)**
A long straight wire along the z-axis carries an oscillating current $I = I_0 \cos \omega t$, generating (at some moment in time, and at some location) a magnetic field as indicated in the figure.



a) Copy this sketch to your answer sheet and add (and label!) the following vectors: wavevector $\vec{k}$, Poynting vector $\vec{S}$, electric field $\vec{E}$.
For a traveling wave, $\vec{B}$, $\vec{E}$ are orthogonal w.r.t. each other, and also orthogonal w.r.t. $\vec{k}$, the propagation vector. The Poynting vector is defined as $\vec{S} = \vec{E} \times \vec{B}$. The symmetry of the problem (long straight wire) dictates that the resulting wave travels outward from the wire, so $\vec{k}$ points away from the wire. Given the direction of $\vec{B}$, the direction $\vec{E}$ can be determined as up.



b) Assume $E(\vec{r}, t) = E_0(\vec{r}) \cos(\vec{k} \cdot \vec{r} - \omega \cdot t)$. Give and explain the position dependence of $E_0(\vec{r})$.
The wave that is generated is a cylindrical wave. The irradiance is proportional to $I \propto E^2$.

The area of a wavefront is proportional to $r = \sqrt{x^2 + y^2}$, so $A \propto r$. The total energy in the wavefront $E = IA \propto Ir$ is conserved, so $I \propto 1/r$, and thus $E \propto 1/\sqrt{r}$

c) Give the expression for the phase velocity v of this wave.
For a traveling wave with $E \propto \cos(\vec{k} \cdot \vec{r} - \omega \cdot t)$ the velocity is given by $v = \omega/|\vec{k}| = \omega/k$.

d) If $\omega = 2\pi \cdot 10^{15}\,\mathrm{s}^{-1}$ and $\lambda = 200\,\mathrm{nm}$, calculate the refractive index of the medium the wave is propagating in.
Use $k = 2\pi/\lambda$, then $v = \omega/k = \omega\lambda/2\pi = 2 \cdot 10^8\,\mathrm{m/s}$. Since $v = c/n$ and $c = 3 \cdot 10^8\,\mathrm{m/s}$, $n = 1.5$.

2. **Dispersion (12 points)**
For low density materials the refractive index can be approximated by

$$n^2(\omega) - 1 = \frac{N q_e^2}{\varepsilon_0 m_e} \sum_j \frac{f_j}{\omega_{0j}^2 - \omega^2 + i\gamma\omega} \tag{1}$$

For high density materials this becomes

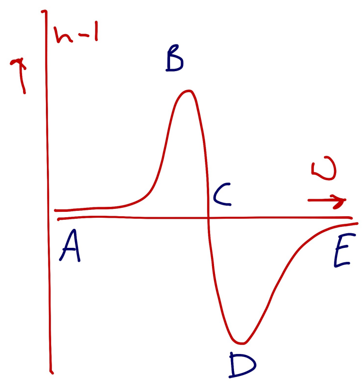
$$\frac{n^2(\omega) - 1}{n^2(\omega) + 2} = \frac{N q_e^2}{3\varepsilon_0 m_e} \sum_j \frac{f_j}{\omega_{0j}^2 - \omega^2 + i\gamma\omega} \tag{2}$$

a) Identify each symbol in (1) or (2), and very briefly explain what is it.
n: refractive index
ω: (angular) frequency of the incident light wave
N: volume density of oscillators/electrons
q_e: (absolute) electron charge
ε_0: electric permittivity of vacuum; m_e: electron mass
ω_{0j}: natural (angular) frequency of oscillator j
f_j: fraction of oscillators with ω_{0j}
i: imaginary unit
γ: damping coefficient.
See also pg. 70 of Hecht.

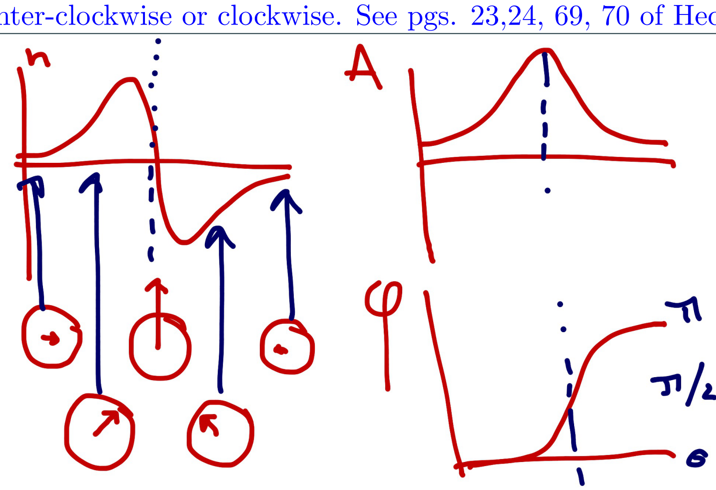
b) Explain why different results are obtained for low and high density materials of otherwise the same properties.
In dense materials the polarization of the material generates a field which is also felt by the oscillators in addition to the incoming external wave. See pg. 71 of Hecht.

c) Give the relation between n, ε, and P (polarization).
$n^2 = \varepsilon/\varepsilon_0$, $\varepsilon = \varepsilon_0 + P/E$. See pg. 66 & 70 of Hecht.

d) Consider the refractive index as sketched below. For the locations A – E, sketch the corresponding phasor for the *secondary* wave. Take the phasor for the *primary* wave to point along the horizontal axis.



The amplitude variation is bell-shaped from left to right (see top right), with maximum at C. The phase shifts between the primary and secondary waves varies from zero for A to 180° for point E. It is 90° for point C, where $\omega = \omega_0$. (see bottom right). The resulting phasors are shown bottom left. Both the length and the angle of the arrow should be visible. The arrow may rotate counter-clockwise or clockwise. See pgs. 23,24, 69, 70 of Hecht.



3. **Planetary Atmosphere (12 points)**

a) Briefly explain the concept of Rayleigh scattering. Include the condition(s) to be met, wavelength dependence, and directional dependence (for a single scatterer, for thin media, and dense media).
Condition: small scatterers, *size* $< \lambda/15$. Instantaneous re-emission; isotropic emission (spherical wave); intensity $\propto 1/\lambda^4$. Thin media: sideways scattering suppressed; thick media: even more suppressed. See pg. 86–92 of Hecht for a detailed discussion.

b) The *transit method* is used by astronomers to study the atmosphere of far away planets. When a planet passes in front of its parent star, the amount of light will temporarily dim (because the planet blocks the star). Also the spectral composition of the light may change. If the planet has an atmosphere, and the star shines through it, will the transmitted light become more blue or more red? Explain.
More red. Since we are far away from the planet, we see forward scattering. Because of the wavelength dependence, blue light ($\lambda \simeq 400\,\text{nm}$) scatters more than red light $\lambda \simeq 800\,\text{nm}$).

Consequently more blue light will be removed from the starlight then red light. Hence the light looks more red.

c) Use Fermat's principle and the decreasing density of the earth's atmosphere with height to explain (e.g. with a sketch) why the sun rises earlier than you would expect from planetary motion alone. Does the sun also set earlier?
Light will bend towards higher refractive index; n is proportional to $\sqrt{N}$, so light will bend towards earth. Observer will see light coming from higher in the sky then where the sun is. So even when the sun is still below the horizon (as predicted by planetay motion), it can still be seen above the horizon. So the sun rises above the horizon earlier than predicted. Along the same arguments, the sun becomes invisible *a while after* the sun has dipped below the horizon. So it sets too late.